

# Direct measurement of $\gamma$ using $B \rightarrow K\pi\pi$ and $B \rightarrow KK\bar{K}$ decays

Bhubanjyoti Bhattacharya and David London

*Physique des Particules, Université de Montréal,  
C.P. 6128, succ. centre-ville, Montréal, QC, Canada H3C 3J7*

Maxime Imbeault

*Département de physique, Cégep de Saint-Laurent,  
625, avenue Sainte-Croix, Montréal, QC, Canada H4L 3X7*

Using the BABAR measurements of the Dalitz plots for  $B^0 \rightarrow K^+\pi^0\pi^-$ ,  $B^0 \rightarrow K^0\pi^+\pi^-$ ,  $B^+ \rightarrow K^+\pi^+\pi^-$ ,  $B^0 \rightarrow K^+K^0K^-$ , and  $B^0 \rightarrow K^0K^0\bar{K}^0$  decays, we cleanly extract the weak phase  $\gamma$ . We find four possible solutions:  $(31^{+2}_{-3})^\circ$ ,  $(77 \pm 3)^\circ$ ,  $(258^{+4}_{-3})^\circ$ , and  $(315^{+3}_{-2})^\circ$ . In all cases the error includes first-order flavor-SU(3) breaking effects. One solution  $-(77 \pm 3)^\circ$  is consistent with the standard model; its error is far smaller than that obtained using two-body  $B$  decays.

PACS numbers: 11.30.Hv, 12.15.Ji, 13.25.Hw, 14.40.Nd

One of the main aims of  $B$  physics is to test the standard model (SM) explanation of CP violation, which is that it is due to a complex phase in the Cabibbo-Kobayashi-Maskawa (CKM) quark mixing matrix. To this end, one measures the three angles of the unitarity triangle [1],  $\alpha$ ,  $\beta$  and  $\gamma$ , in many different ways, and looks for discrepancies.

The standard method to directly probe  $\gamma$  was originally suggested by Gronau, London, and Wyler (GLW) [2], with variants subsequently proposed by Atwood, Dunitz and Soni [3], and by Giri, Grossman, Soffer and Zupan [4]. It involves the measurement of the rates for the two-body decays  $B^\pm \rightarrow DK^\pm$ , where  $D = D^0$ ,  $\bar{D}^0$  or  $D_{CP} \equiv (D^0 \pm \bar{D}^0)/\sqrt{2}$  ( $D_{CP}$  decays into a CP eigenstate). Although the two-body method is expected to be theoretically clean, it is difficult experimentally, so that the present direct measurement, using a combination of the techniques of Refs. [2–4], has a large error:  $\gamma = (66 \pm 12)^\circ$  [5].

Recently it was shown that, contrary to previous thinking, it is possible to extract CKM phase information from charmless three-body  $B$  decays [6, 7]. There are three ingredients. First, the three-body decay amplitudes can be expressed in terms of diagrams similar to those of two-body  $B$  decays [8]. These are described in Ref. [6]. (As we consider  $\bar{b} \rightarrow \bar{s}$  transitions, the  $B^+$  decay amplitude can receive a contribution from the annihilation diagram. This is neglected.) Note that, unlike the two-body diagrams, the three-body diagrams are momentum dependent.

Second, it is possible to fix the symmetry of the final state. This is done using the Dalitz plot of  $B \rightarrow P_1P_2P_3$  (the  $P_i$  are pseudoscalar mesons) [6]. Denoting by  $p_i$  the momentum of each  $P_i$ , one defines the three Mandelstam variables  $s_{ij} \equiv (p_i + p_j)^2$ . These are not independent, but obey  $s_{12} + s_{13} + s_{23} = m_B^2 + m_1^2 + m_2^2 + m_3^2$ . Now, the Dalitz plot is given in terms of two Mandelstam variables, say  $s_{12}$  and  $s_{13}$ . The key point is that the experimental Dalitz-plot analysis allows one to reconstruct the decay

amplitude  $\mathcal{M}(B \rightarrow P_1P_2P_3)(s_{12}, s_{13})$ . The amplitude for a state with a given symmetry is then found by applying this symmetry to  $\mathcal{M}(s_{12}, s_{13})$ . This amplitude is used to compute all (momentum-dependent) observables for the decay. For example, the final state  $K_S\pi^+\pi^-$  has CP + if the  $\pi^+\pi^-$  pair is symmetrized. The amplitude for this state is  $[\mathcal{M}(s_{12}, s_{13}) + \mathcal{M}(s_{13}, s_{12})]/\sqrt{2}$ .

Third, in Ref. [7] it was shown that, as is the case in two-body decays [9], under flavor SU(3) there are relations between the electroweak penguin (EWP) and tree diagrams for  $\bar{b} \rightarrow \bar{s}$  transitions. These take the simple form  $P'_{EWi} = \kappa T'_i$  and  $P'^C_{EWi} = \kappa C'_i$  ( $i = 1, 2$ ), where

$$\kappa \equiv -\frac{3}{2} \frac{|\lambda_t^{(s)}|}{|\lambda_u^{(s)}|} \frac{c_9 + c_{10}}{c_1 + c_2}, \quad (1)$$

in which the  $c_i$  are Wilson coefficients and  $\lambda_p^{(s)} = V_{pb}^* V_{ps}$ . Note: the EWP-tree relations hold only for the state that is fully symmetric under exchanges of the final-state particles. However, the amplitude for this state can be found as described above using the Dalitz plot:

$$\mathcal{M}_{\text{fs}} = \frac{1}{\sqrt{6}} [\mathcal{M}(s_{12}, s_{13}) + \mathcal{M}(s_{13}, s_{12}) + \mathcal{M}(s_{12}, s_{23}) + \mathcal{M}(s_{23}, s_{12}) + \mathcal{M}(s_{23}, s_{13}) + \mathcal{M}(s_{13}, s_{23})], \quad (2)$$

where the subscript “fs” stands for “fully symmetric.”

Based on the above, a method was proposed for extracting the weak phase  $\gamma$  from  $B \rightarrow K\pi\pi$  and  $B \rightarrow KK\bar{K}$  decays [10]. It works as follows. The following five decays are considered:  $B^0 \rightarrow K^+\pi^0\pi^-$ ,  $B^0 \rightarrow K^0\pi^+\pi^-$ ,  $B^+ \rightarrow K^+\pi^+\pi^-$ ,  $B^0 \rightarrow K^+K^0K^-$ , and  $B^0 \rightarrow K^0K^0\bar{K}^0$ . In writing the amplitudes of these five processes in terms of diagrams, we note the following. In three-body decays, one has to “pop” a quark pair from the vacuum. For  $B \rightarrow K\pi\pi$  decays, the popped quark pair is  $u\bar{u}$  or  $d\bar{d}$  (under isospin, these diagrams are equal), while the  $B \rightarrow KK\bar{K}$  decays may have a popped  $s\bar{s}$  pair. Now, the imposition of the EWP-tree relations assumes

flavor SU(3) symmetry. But this also implies that diagrams with a popped  $s\bar{s}$  quark pair are equal to those with a popped  $u\bar{u}$  or  $d\bar{d}$ . In other words, under flavor-SU(3) symmetry the diagrams in  $B \rightarrow K K \bar{K}$  decays are the same as those in  $B \rightarrow K \pi \pi$  decays.

Note, however, that flavor-SU(3) symmetry is not exact. It is therefore important to keep track of a possible difference between  $B \rightarrow K \pi \pi$  and  $B \rightarrow K K \bar{K}$  decays.

The amplitudes for the five decays in terms of diagrams are given in Ref. [10]. We define the following four effective diagrams:

$$\begin{aligned} a &\equiv -\tilde{P}'_{tc} + \kappa \left( \frac{2}{3}T'_1 + \frac{1}{3}C'_1 + \frac{1}{3}C'_2 \right), \\ b &\equiv T'_1 + C'_2, \quad c \equiv T'_2 + C'_1, \quad d \equiv T'_1 + C'_1. \end{aligned} \quad (3)$$

The decay amplitudes can now be written in terms of five diagrams,  $a$ - $d$  and  $\tilde{P}'_{uc}$ :

$$\begin{aligned} 2A(B^0 \rightarrow K^+\pi^0\pi^-)_{\text{fs}} &= be^{i\gamma} - \kappa c, \\ \sqrt{2}A(B^0 \rightarrow K^0\pi^+\pi^-)_{\text{fs}} &= -de^{i\gamma} - \tilde{P}'_{uc}e^{i\gamma} - a + \kappa d, \\ \sqrt{2}A(B^+ \rightarrow K^+\pi^+\pi^-)_{\text{fs}} &= -ce^{i\gamma} - \tilde{P}'_{uc}e^{i\gamma} - a + \kappa b, \\ \sqrt{2}A(B^0 \rightarrow K^+K^0K^-)_{\text{fs}} &= \alpha_{SU(3)} \times \\ &\quad (-ce^{i\gamma} - \tilde{P}'_{uc}e^{i\gamma} - a + \kappa b), \\ A(B^0 \rightarrow K^0K^0\bar{K}^0)_{\text{fs}} &= \alpha_{SU(3)}(\tilde{P}'_{uc}e^{i\gamma} + a), \end{aligned} \quad (4)$$

where  $\alpha_{SU(3)}$  measures the amount of flavor-SU(3) breaking.

In the flavor-SU(3) limit ( $|\alpha_{SU(3)}| = 1$ ),  $A(B^+ \rightarrow K^+\pi^+\pi^-)_{\text{fs}} = A(B^0 \rightarrow K^+K^0K^-)_{\text{fs}}$ , so that the  $B^+$  decay does not furnish any new information. The remaining four amplitudes depend on ten theoretical parameters: five magnitudes of diagrams, four relative strong phases, and  $\gamma$ . But there are eleven experimental observables: the decay rates and direct CP asymmetries for the four  $B^0$  decays, and the indirect CP asymmetries of  $B^0 \rightarrow K^0\pi^+\pi^-$ ,  $B^0 \rightarrow K^+K^0K^-$  and  $B^0 \rightarrow K^0K^0\bar{K}^0$ . With more observables than theoretical parameters,  $\gamma$  can be extracted from a fit. Furthermore, if one allows for SU(3) breaking ( $|\alpha_{SU(3)}| \neq 1$ ), we can add two more observables: the decay rate and direct CP asymmetry for the  $B^+$  decay. In this case it is possible to extract  $\gamma$  even with the inclusion of  $|\alpha_{SU(3)}|$  as a fit parameter.

As has been stressed above, the diagrams and observables are both momentum dependent. That is, the above method for extracting  $\gamma$  in fact applies to each point in the Dalitz plot. However, the value of  $\gamma$  is independent of momentum, so that the method really represents *many* independent measurements of  $\gamma$ . A preliminary analysis presented in Ref. [11] considered a naive average over such independent measurements of  $\gamma$ . In this letter we improve upon this and perform a combined likelihood fit to extract  $\gamma$  from multiple Dalitz-plot points.

The first step in performing a fit is to collect the observables. This is done as follows. An isobar model is

used to analyze the three-body Dalitz plots. Here the decay amplitude is expressed as the sum of a non-resonant and several intermediate resonant contributions:

$$\mathcal{M}(s_{12}, s_{13}) = \mathcal{N}_{\text{DP}} \sum_j c_j e^{i\theta_j} F_j(s_{12}, s_{13}), \quad (5)$$

where the index  $j$  runs over all contributions. Each contribution is expressed in terms of isobar coefficients  $c_j$  (amplitude) and  $\theta_j$  (phase), and a dynamical wave function  $F_j$ .  $\mathcal{N}_{\text{DP}}$  is a normalization constant. The  $F_j$  take different forms depending on the contribution. The  $c_j$  and  $\theta_j$  are extracted from a fit to the Dalitz-plot event distribution. Such fits were performed by BABAR [12] for each of the five three-body decays of interest. The isobar coefficients found, together with their assumed wave functions ( $F_j$ ), allow us to reconstruct the amplitude for each three-body decay as a function of the relevant Mandelstam variables. We have chosen the normalization constant such that the integral of  $|\mathcal{M}|^2$  over the kinematically-allowed Dalitz-plot phase space gives the experimental branching fraction ( $\mathcal{B}_{\text{Exp}}$ ). We then construct  $\mathcal{M}_{\text{fs}}$  using Eq. (2). This process is repeated with the information available for the CP-conjugate process, where we construct  $\overline{\mathcal{M}}_{\text{fs}}$ .

The experimental observables are then obtained as follows:

$$\begin{aligned} X(s_{12}, s_{13}) &= |\mathcal{M}_{\text{fs}}(s_{12}, s_{13})|^2 + |\overline{\mathcal{M}}_{\text{fs}}(s_{12}, s_{13})|^2, \\ Y(s_{12}, s_{13}) &= |\mathcal{M}_{\text{fs}}(s_{12}, s_{13})|^2 - |\overline{\mathcal{M}}_{\text{fs}}(s_{12}, s_{13})|^2, \\ Z(s_{12}, s_{13}) &= \text{Im} [\mathcal{M}_{\text{fs}}^*(s_{12}, s_{13}) \overline{\mathcal{M}}_{\text{fs}}(s_{12}, s_{13})]. \end{aligned} \quad (6)$$

In order to obtain the experimental error bars on these quantities we allow the input isobar coefficients to vary over their  $1\sigma$  allowed ranges. Note that, although the errors on the isobar coefficients extracted from a given Dalitz plot are in general correlated, such information is not always publicly available. In our analysis we have considered the errors to be completely uncorrelated, but we hope that a future analysis by an experimental collaboration will take such effects into account.

The effective CP-averaged branching ratio ( $X$ ), direct CP asymmetry ( $Y$ ), and indirect CP asymmetry ( $Z$ ) may be constructed for every point on any Dalitz plot. However, when the final state has a specific flavor, such as in the case of  $B^0 \rightarrow K^+\pi^0\pi^-$ , the quantity  $Z$  has no physical meaning and is therefore left out of our analysis. In addition, note that, since the amplitudes used to construct these observables are fully symmetric under the interchange of the three Mandelstam variables, for any given point on a Dalitz plot there will be five other points where the extracted  $X$ ,  $Y$  and  $Z$  take identical values, and hence do not provide any new information. In order to avoid counting the same information multiple times, we therefore divide each Dalitz plot into six zones by its three axes of symmetry, and use information only

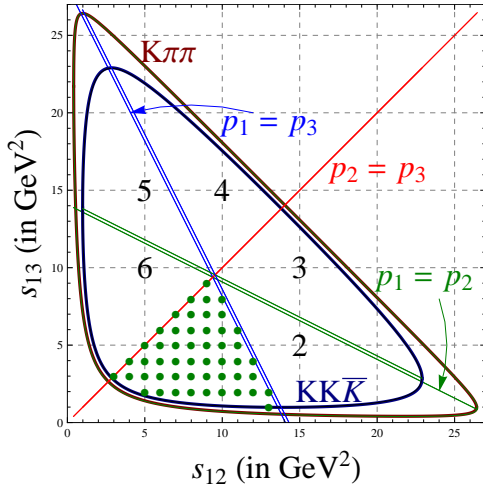


FIG. 1: Kinematic boundaries and symmetry axes of  $B \rightarrow K\pi\pi$  and  $B \rightarrow KK\bar{K}$  Dalitz plots. The symmetry axes divide each plot into six zones, five of which are marked 2-6. The fifty dots in the region of overlap of the first of six zones from all Dalitz plots are used for the  $\gamma$  measurement.

from one zone. This is shown in Fig. 1, where we select the dotted zone for our calculations.

Although it is possible to measure both the direct and indirect CP asymmetries in  $B^0 \rightarrow K_S K_S K_S$ , their measurement is currently statistics limited. The experimental Dalitz-plot analysis done by BABAR makes no distinction between the amplitude and its CP conjugate. That is, they take  $A(B^0 \rightarrow K_S K_S K_S) = A(\bar{B}^0 \rightarrow K_S K_S K_S)$ . This has two consequences. First,  $Y$  and  $Z$  vanish for every point of the Dalitz plot. Second, this requires that  $\tilde{P}'_{uc}$  be set to zero in Eq. (4). The removal of an equal number of unknown parameters (amplitude and phase of  $\tilde{P}'_{uc}$ ) and observables does not affect the viability of the method described above.

With the observables in hand, we now perform a maximum likelihood analysis for extracting  $\gamma$ . We select points within a sixth of the overlapping kinematically-allowed region of the Dalitz plots separated from one another by  $1 \text{ GeV}^2$  in both Mandelstam variables. We find that there are fifty such points. (Note that the  $1 \text{ GeV}^2$  resolution is ad hoc. In a more appropriate analysis, one should consider the optimal bin size for the Dalitz plot with the lowest available statistics to determine this resolution.) For each of these fifty points we construct the  $-2\Delta \ln L(\gamma)$  function, where  $L$  represents the likelihood, which we then minimize over all the hadronic parameters for that point. The sum of such functions over all fifty points gives us a joint likelihood distribution. The local minima of this function are then identified as the most-likely values of  $\gamma$ . In order to find the  $1\sigma$  error bar on  $\gamma$  we first shift the likelihood function along the vertical axis so that the zero of the function corresponds to a local minimum. We then look for the range of  $\gamma$  that results in a unit shift along the vertical axis of the  $-2\Delta \ln L(\gamma)$

vs  $\gamma$  plot.

We perform the likelihood maximization fit in three different ways and plot our results in Fig. 2. We first consider the scenario in which flavor SU(3) is a good symmetry. That is, we fix  $|\alpha_{SU(3)}| = 1$ ; our analysis involves only the four  $B^0$  decay channels. The most-likely values of  $\gamma$  obtained in this way are listed under Fit 1 in Table I.

Second, we allow for SU(3) breaking and treat it as follows. We compare the Dalitz plots for the two processes  $B^+ \rightarrow K^+ \pi^+ \pi^-$  and  $B^0 \rightarrow K^+ K^0 K^-$  point by point. Theoretically, the amplitudes for these processes differ only by the parameter  $\alpha_{SU(3)}$ . The ratio of  $X$ 's constructed from the two Dalitz plots then gives us  $|\alpha_{SU(3)}|^2$ . (Note that a similar ratio constructed from the  $Y$ 's has an enormous error due to the smallness of  $Y$ . We are therefore unable to extract any interesting physical information from such a ratio.) Averaged over the fifty points we find  $|\alpha_{SU(3)}| = 0.97 \pm 0.05$ . This shows that, on average, SU(3) breaking is small. We use  $|\alpha_{SU(3)}|$  found in this way to correct the observables from the  $B \rightarrow KK\bar{K}$  Dalitz plots and use the corrected numbers in a new maximum-likelihood analysis for finding  $\gamma$ . We present the results under Fit 2 in Table I.

In the third maximum-likelihood analysis, we consider observables from all five Dalitz plots but now include  $|\alpha_{SU(3)}|$  as an additional unknown hadronic parameter. The results from this method are listed under Fit 3 in Table I.

TABLE I: Most likely values of  $\gamma$  (in degrees) extracted from Fig. 2. Results are obtained using the three different fitting methods as explained in the text.

Solution	Fit 1	Fit 2	Fit 3
I	$31^{+3}_{-2}$	$31 \pm 2$	$31^{+2}_{-3}$
II	$76^{+3}_{-2}$	$78^{+2}_{-3}$	$77 \pm 3$
III	$261^{+2}_{-4}$	$259 \pm 3$	$258^{+4}_{-3}$
IV	$314 \pm 2$	$315 \pm 2$	$315^{+3}_{-2}$

The maximum-likelihood analysis indicates that, in each of the three methods described above, the data favor four distinct discretely-ambiguous values of  $\gamma$ . In Table I we present the most-likely values of  $\gamma$  extracted using these three methods. It is evident from the results that the inclusion of an SU(3)-breaking parameter  $|\alpha_{SU(3)}|$  shifts the preferred values of  $\gamma$  by only a tiny amount. This indicates that the leading-order effects of flavor-SU(3) breaking are well under control in three-body  $B$  decays. While one cannot completely remove this source of theoretical error from our analysis, the error bars are rather small.

Even though there are four preferred values of  $\gamma$ , in all cases the error is small,  $2$ - $4^\circ$ . Although this may seem surprising at first sight, it really is not when one remem-

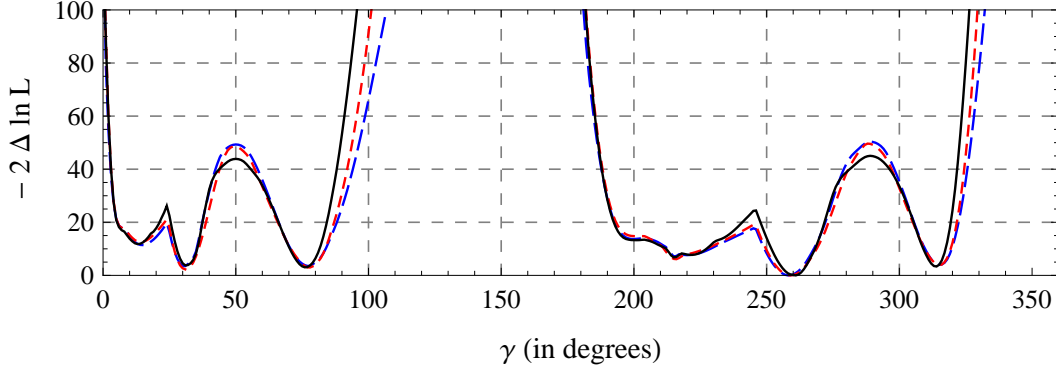


FIG. 2: Results of maximum-likelihood fits. The solid (black) curve represents the fit assuming flavor-SU(3) symmetry. The short dashes (red) represent the fit where flavor-SU(3) breaking is fixed by a point-by-point comparison of Dalitz plots for  $B^+ \rightarrow K^+ \pi^+ \pi^-$  and  $B^0 \rightarrow K^+ K^0 K^-$ . The long dashes (blue) represent the fit with inputs from five Dalitz plots and an extra hadronic fit parameter  $|\alpha_{SU(3)}|$ .

bers that there are, in fact, fifty independent measurements of  $\gamma$ . Roughly speaking, if each measurement has an error of  $\pm 20^\circ$ , which is somewhat larger than other methods, then when we take a naive average, we divide the error by  $\sqrt{50}$ , giving a final error of  $\sim 3^\circ$ . The one caveat is that all errors considered to this point have been entirely statistical – the systematic error has not been included (we do not know its value). Hopefully, the experimentalists themselves will redo this analysis, including all errors.

There are other sources of error that have not been included in our method. First, as mentioned above, we have assumed that the errors on the isobar coefficients are uncorrelated. In practice such errors are correlated, which will alter the error bars on  $X$ ,  $Y$  and  $Z$ . Second, we have only taken leading-order flavor-SU(3) breaking into account. Higher-order flavor-SU(3) breaking may arise due to the nonzero mass difference between pions and kaons, and between intermediate resonances. This said, the error due to leading-order SU(3) breaking is evidently small. It is unlikely that the error due to higher-order SU(3) breaking is larger.

Finally, we reiterate that we find four most-likely values of  $\gamma$  in  $B \rightarrow K\pi\pi$  and  $B \rightarrow KK\bar{K}$  decays. Three of these –  $(31^{+2}_{-3})^\circ$ ,  $(258^{+4}_{-3})^\circ$ , and  $(315^{+3}_{-2})^\circ$  – are in disagreement with the SM (is this a “ $K\pi\pi-KK\bar{K}$  puzzle”?). However, one solution –  $(77 \pm 3)^\circ$  – is consistent with the SM. Furthermore, its error (which includes SU(3) breaking, but not other systematic effects) is far smaller than that found with the GLW method, which uses two-body  $B$  decays. The fundamental reason is that three-body decays give many independent measurements of  $\gamma$ . These can be combined, reducing the error on  $\gamma$ .

A special thank you goes to E. Ben-Haim for his important input to this project. We also thank J. Charles, M. Gronau, N. Rey-Le Lorier, J. Rosner and J. Smith for helpful communications. BB would like to thank G.

Bell and WG IV of CKM 2012. This work was facilitated in part by the workshop *New Physics from Heavy Quarks in Hadron Colliders*, which was sponsored by the University of Washington and supported by the DOE under contract DE-FG02-96ER40956. This work was financially supported by NSERC of Canada (BB, DL) and by FQRNT du Québec (MI).

- 
- [1] J. Beringer *et al.* [Particle Data Group Collaboration], Phys. Rev. D **86**, 010001 (2012).
  - [2] M. Gronau and D. London, Phys. Lett. B **253**, 483 (1991); M. Gronau and D. Wyler, Phys. Lett. B **265**, 172 (1991).
  - [3] D. Atwood, I. Dunietz and A. Soni, Phys. Rev. Lett. **78**, 3257 (1997) [hep-ph/9612433], Phys. Rev. D **63**, 036005 (2001) [hep-ph/0008090].
  - [4] A. Giri, Y. Grossman, A. Soffer and J. Zupan, Phys. Rev. D **68**, 054018 (2003) [hep-ph/0303187].
  - [5] J. Charles *et al.* [CKMfitter Group Collaboration], Eur. Phys. J. C **41**, 1 (2005) [hep-ph/0406184], updated results and plots available at <http://ckmfitter.in2p3.fr/>.
  - [6] N. Rey-Le Lorier, M. Imbeault and D. London, Phys. Rev. D **84**, 034040 (2011) [arXiv:1011.4972 [hep-ph]].
  - [7] M. Imbeault, N. Rey-Le Lorier and D. London, Phys. Rev. D **84**, 034041 (2011) [arXiv:1011.4973 [hep-ph]].
  - [8] M. Gronau, O. F. Hernandez, D. London and J. L. Rosner, Phys. Rev. D **50**, 4529 (1994), Phys. Rev. D **52**, 6374 (1995).
  - [9] M. Neubert and J. L. Rosner, Phys. Lett. B **441**, 403 (1998) [arXiv:hep-ph/9808493], Phys. Lett. B **441**, 403 (1998) [arXiv:hep-ph/9808493]; M. Gronau, D. Pirjol and T. M. Yan, Phys. Rev. D **60**, 034021 (1999) [Erratum-ibid. D **69**, 119901 (2004)] [arXiv:hep-ph/9810482].
  - [10] N. Rey-Le Lorier and D. London, Phys. Rev. D **85**, 016010 (2012) [arXiv:1109.0881 [hep-ph]].
  - [11] B. Bhattacharya, M. Imbeault and D. London, arXiv:1212.1167 [hep-ph].
  - [12]  $B^0 \rightarrow K^+ \pi^0 \pi^-$ : J. P. Lees *et al.* [BABAR Collabora-

tion], Phys. Rev. D **83**, 112010 (2011) [arXiv:1105.0125 [hep-ex]];  $B^0 \rightarrow K^0 \pi^+ \pi^-$ : B. Aubert *et al.* [BABAR Collaboration], Phys. Rev. D **80**, 112001 (2009) [arXiv:0905.3615 [hep-ex]];  $B^+ \rightarrow K^+ \pi^+ \pi^-$ : B. Aubert *et al.* [BABAR Collaboration], Phys. Rev. D **78**, 012004 (2008) [arXiv:0803.4451 [hep-ex]];  $B^0 \rightarrow K^+ K^0 K^-$ :

J. P. Lees *et al.* [BABAR Collaboration], Phys. Rev. D **85**, 112010 (2012) [arXiv:1201.5897 [hep-ex]];  $B^0 \rightarrow K^0 K^0 \bar{K}^0$ : J. P. Lees *et al.* [BABAR Collaboration], Phys. Rev. D **85**, 054023 (2012) [arXiv:1111.3636 [hep-ex]].